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HIGHEST COMMON FACTOR.

By Howard F. Hart.

As the colleges now are requiring highest common factor by factoring methods only, any plan whereby the number of factors to be tried can be lessened is certainly worth while. For in general we must regard as possible any binomial factor whose first-degree term is an integral divisor of the highest term of the given expression and whose independent term is an integral divisor of the independent term of the expression. Thus in such a problem as, "Find the H.C.F. of $5x^3-21x^2+5x-4$ and $5x^3-19x^2-5x+4$" (McCurdy's Exercise Book, page 40, example 4) the possible factors that a student might try and must try, unless he were very lucky in those he chose to try first, are $x\pm 1$, $x\pm 2$, $x\pm 4$, $5x\pm 1$, $5x\pm 2$, $5x\pm 4$. And further if the given expressions were, say, cubics having no common binomial factor at all but with a quadratic one instead (*e. g.*, $2x^3+5x^2+x-3$ and $2x^3-x^2-5x+3$) I doubt if the ordinary first-year student would get any result unless it were unity.

As definite methods in eliminating factors and thus reducing the labors of factoring by trial I have found the following to be very useful as engines in attack. With one small exception (viz., a cubic having always at least one real root) even the proofs are not above what a first-year student can grasp.

First, if any one of the given expressions is a quadratic, that, or either of its factors, or unity are the only possibilities. And as quadratics are readily factored by inspection, if they can be factored at all, there is no real problem in highest common factor unless the given expressions are all at least cubics.

Sometimes one of the given expressions can be readily factored even though the others cannot. In such a case there is no real problem either as it is merely a question of trying the factors of the factored expression. And as we shall see in a moment we might not have to try all of those.

So our problem is: Given two (in case three or more expressions are given we take them two at a time) cubics or higher forms neither of which can be readily factored to find the product of their common factors.

In the first place we should note that if a cubic can be factored at all there will be one binomial factor (corresponding to its necessarily one real root).

And in the second place it is obvious that in cases where the highest terms differ in coefficients or the lowest terms are different that certain apparently possible factors of each expression cannot be common factors and hence need not be tried. Thus in the problem "$x^5 - 2x^4 + x^2$ and $2x^4 - 4x^3 - 4x + 6$" (Sheffield) $x \pm 2$, $x \pm 3$, $x \pm 6$, $2x \pm 1$, $2x \pm 3$, $2x \pm 6$ cannot be common factors. Therefore all binomial factors but $x \pm 1$ or $(2x \pm 2)$ have been excluded without the labor of trial. Regarded as a point in division even the dullest students appreciate, it will be found, *why* such factors can be excluded at once.

These two methods between them often reduce the problem to the consideration of one binomial factor.

Still there are cases where, unless the student can in general factor at least one of the given forms completely he never knows but that there might be other common factors which he has not found.

In this connection the theorem, "Any *common* factor of two quantities is a factor of the sum and difference of the given quantities or multiples of them" is of great aid, for often the sum or difference is a vastly simpler function than either of the given ones and readily factorable. And its proof is so simple that first-year students can comprehend it—particularly if it is taken up as a generalization of many previous tested cases.

Let E_1 and E_2 represent the expressions, f the product of their common factors so that

$$\begin{array}{lr} & aE_1 = afX \\ \text{and} & bE_2 = bfY \\ \text{then} & aE_1 \pm bE_2 = f(aX \pm bY) \end{array}$$

where X and Y represent the products of the remaining factors.

The same special cases will make it clear that the converse is unfortunately not a true theorem.

To illustrate its use suppose we apply it first to the problems given above in the first paragraph. In the first one on adding we have $10x^3-40x^2$ or $10x^2(x-4)$ and as we know that the desired result is the product of the common factors only and that every factor common to the given expressions is a factor of $10x^3-40x^2$ we merely have to discover whether $x-4$ is a common factor or not.

In the second example addition of the forms gives $4x(x^2+x-1)$ and as x^2+x-1 is discovered by trial to be a common factor it is therefore by the same logic as above the H.C.F.

Of course wherever H.C.F. is needed this method will apply. Thus, suppose we are asked to reduce $\frac{x^3-5x^2-2x+24}{x^3-4x^2-3x+18}$ to lowest terms. If we subtract we obtain $-(x^2-x-6)$ or $(-1)(x-3)(x+2)$ and trying both $x-3$ and $x+2$ as divisors we get as a solution, $\frac{x-4}{x-3}$.

And again. If the relation $\frac{E_1E_2}{\text{H.C.F.}}=\text{L.C.M.}$ is used as the method of finding the L.C.M. that problem becomes merely a problem in H.C.F. and therefore as readily solved.

Finally we might note that if the given expressions have unlike coefficients it is sometimes possible to get a simple sum or difference by taking multiples as, *e. g.*, "To find the H.C.F. of $4a^5+14a^4+20a^3+70a^2$ and $6a^6+21a^5-12a^4-43a^3$" (Robbins and Somerville Exercise Book, page 42, example 11). If we remove the common factor a^2, multiply the first by $3a$, the second by 2, and subtract we have $42a(2a+7)$. On trial we find that $(2a+7)$ is a common factor so that the solution is $a^2(2a+7)$.

In all of the above cases the trial of the suggested factors is best made by synthetic division for an exposition of which read Mr. E. R. Smith's article in the MATHEMATICS TEACHER for September, 1910.

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